

Seminario de Análisis Numérico y Modelamiento Matemático de Estudiantes

Discretizing Hodge-Dirac Equations in the Framework of Finite Element Systems

The Dirac equation, conceived by Paul Dirac in 1928, is formulated as a relativistic generalization of Schrödinger's equation, aiming to describe the evolution of the quantum wave function in a relativistically consistent way. It is closely related to an equation posed in the exterior bundle: the so-called Hodge-Dirac equation. In this talk, we show how a Dirac equation in Minkowski space with minimal coupling to an electromagnetic field can be interpreted as a compact perturbation of the eigenmode problem associated with the Hodge-Dirac operator on the exterior bundle of $\mathbf{R}^3$. Our approach is strongly influenced by [LS16, Chr18]. We develop an abstract framework for the Galerkin discretization of (non-definite) symmetric Fredholm operators in Hilbert spaces, and conclude with an application of this theory to the aforementioned problem posed in an n -dimensional flat torus, relying on the framework of finite element systems, as coined by Snorre H. Christiansen in [Chr08]; the choice of domain is made so as to avoid boundary conditions and curvature terms in the formulation.

References

- [Chr08] Snorre H. Christiansen, A construction of spaces of compatible differential forms on cellular complexes, *Math. Models Methods Appl. Sci.* 18 (2008), no. 5, 739-757.
[Chr18] Snorre H. Christiansen, On eigenmode approximation for Dirac equations: Differential forms and fractional Sobolev spaces, *Math. Com.* 87 (2018), no. 1, 547-580.
[LS16] Paul Leopardi and Ari Stern, The abstract Hodge-Dirac operator and its stable discretization, *SIAM J. Numer. Anal.* 54 (2016), no. 6, 3258-3279.

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